

# Control of the boiling crisis: analysis of a model system

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Received 25 July 2007 / Received in final form 9 October 2007

Published online 29 November 2007 – © EDP Sciences, Società Italiana di Fisica, Springer-Verlag 2007

**Abstract.** Controlling the transition between the low (nucleate) and high temperature (film) regimes of boiling is a serious challenge for a number of technological applications. Based on the theoretical analysis of a simplified reaction-diffusion model, it has recently been shown [A. Pumir, V.V. Barelko, *Chaos* **12**, 610 (2002)] that the transition towards the dangerous situation where the high temperature phase tends to invade the whole system requires a higher power in a periodically spatially modulated system than in an homogeneous system. We show here that the transition mechanisms between the various boiling regimes depend on the ratio between the periodicity length along the wire and the characteristic thermal diffusion length. We analyse theoretically a simple experimental setup aimed at testing these ideas. The heater consists of a thin wire, with an applied electric current, with alternatively low resistance and high resistance sections. We determine the gain in stability for a set of realistic values of the parameters.

**PACS.** 64.70.Fx Liquid-vapor transitions – 05.45.-a Nonlinear dynamics and chaos

## 1 Introduction

In several key industrial processes, involving heat-generating elements (HGE), the boiling crisis, or transition between a low-temperature and a high-temperature regimes [1], represents a very significant safety concern. In the low temperature regime, known as the nucleate regime, the heat released from the HGE leads to the production of small vapor bubbles, which are driven away from the heater. These bubbles carry away heat very efficiently, thus maintaining the temperature at a relatively low value. On the other hand, at high temperature, one observes the so-called film regime, corresponding to a solid covered by a thin film of vapour, with a low heat conductivity. As a result, the heat generated remains in the close neighborhood of the HGE, and the resulting temperature rises to very high values, which may lead to the melt-down of the HGE, with possibly dire consequences. This is a very serious problem, in particular for nuclear reactors.

Despite these important technological implications, the physics governing the boiling crisis remains completely understood. How heat is carried away from the HGE depends on subtle processes involving nucleation and growth of vapour bubbles. We use here a very simplified model, based on a description of the temperature of the HGE which reduces the complexity of the heat transfer rate from the HGE,  $q_-$ , to a function of the temperature of the HGE. Such a description in terms of a

reaction-diffusion model has been used to describe semi-quantitatively a number of phenomena [2–5]. The heat production term,  $q_+$ , is the control parameter. Over a range of values of  $q_+$ , the model is bistable, with a low temperature (nucleate boiling) and a high temperature (film boiling) fixed point. In an extended system, there exists a critical value of  $q_+$ ,  $q_c$ , above (below) which a front separating the low from the high-temperature phases will tend to invade the low (high) temperature phase. The front velocity simply goes through 0 at  $q_+ = q_c$ . Based on this observation, the nucleate boiling film is stable provided  $q_+ \leq q_c$ , unstable otherwise.

The way to increase the heat input,  $q_+$ , of the wire, put forward in [6], consists in introducing a spatial modulation of the HGE. Theoretically, it is known that in a system with periodic modulations, fronts separating the two phases have a zero velocity over a range of control parameters [7], see also [8–10]. By alternating periodically active and passive elements, respectively generating much heat, and little or no heat, the front solutions separating the low (nucleate boiling) and high (film boiling) temperature phases stall over a finite interval of values of  $q_+$ :  $\bar{q}_{+,n} \leq q_+ \leq \bar{q}_{+,f}$ .

The theoretical analysis of the resulting system reveals that two characteristic length scales are involved in this problem: the typical length of the active and passive parts of the HGE,  $l$ , on one hand, and the diffusion length scale for temperature,  $\mathcal{L}_T$ , on the other hand. The problem was investigated in [6] mostly in the case where  $\mathcal{L}_T \gtrsim l$ .

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In this paper, we propose a possible experimental realisation of the ideas put forward in [6]. The experimental system consists of a wire, alternating pieces of metals with very different conductivity: copper with a low resistance, and a nickel-chromium alloy, with a much higher resistance. Heat is produced by the Joule effect, induced by an electric current. Realistic values of  $\mathcal{L}_T$  for this system are quite small. In practice, any experimental setup would correspond to the case where  $\mathcal{L}_T \lesssim l$ . We therefore reconsider the problem in this limit, and based on our simple model, we compute the range of values of  $q_+$  over which a front separating a high and a low temperature phase stalls.

In Section 2, we present briefly the theoretical model, and its experimental realisation. The theoretical analysis of the possible transition regimes, as well as the results concerning the stability limits of the system are shown in Section 3. Our concluding remarks are presented in Section 4.

## 2 Model problem

### 2.1 Formulation of the problem

Assuming that the rate of heat removal from the HGE is a simple function of temperature, the equation of evolution for the temperature,  $T(x, t)$ , along a wire in a 1-dimensional geometry is derived from a simple energy balance argument:

$$c\rho \frac{\partial T}{\partial t} = \lambda \frac{\partial^2 T}{\partial x^2} + (q_+ - q_-) \quad (1)$$

where  $c$  is the heat capacity of the wire,  $\rho$  its density,  $\lambda$  its thermal conductivity and  $d$  its diameter.

In the film boiling regime, the heat removal rate is proportional to the difference between the temperature of the HGE,  $T$ , and the temperature of the fluid far away,  $T_0$ :  $q_- = h(T)(T - T_0)$ . The proportionality constant,  $h$ , depends on the precise nature of the boiling phase:  $h_n \approx 3 \text{ W cm}^{-3} \text{ K}^{-1}$  in the nucleate phase, and  $h_f \approx 0.3 \text{ W K}^{-1} \text{ cm}^{-3}$  in the film regime. The transition between the two regimes occurs at a temperature  $T^*$  of order 120–130 °C. As in [6], we simply use an interpolation of  $h$  in the following from:

$$h(T) = [h_f + (h_n - h_f)/(1 + (T/T^*)^p)] \quad (2)$$

where  $p$  is a large number (we take here  $p = 10$ ).

With a current  $I$  flowing through a wire of resistivity  $r$  and diameter  $d$ , the power released per unit length of wire is:

$$q_+ = I^2 \frac{4r}{c\rho\pi d^2} \quad (3)$$

where  $r$ ,  $c$ ,  $\rho$  and  $d$  are the local values of the resistivity, heat capacity, density and diameter of the wire.

Dividing equation (1) throughout by  $(c\rho)$ , one obtains:

$$\frac{\partial T}{\partial t} = D \frac{\partial^2 T}{\partial x^2} + (Q_+ - Q_-) \quad (4)$$

where  $D = \lambda/(c\rho)$  is the diffusion coefficient,  $Q_+ = I^2 \times A$ , where  $A = 4r/(c\rho\pi d^2)$  and  $Q_- = (T - T_0) \times [\alpha_f + (\alpha_n - \alpha_f)/(1 + (T/T^*)^p)]$ , with  $\alpha_{f,n} = h_{f,n}/(c\rho)$ .

In a wire of diameter  $d = 0.01$  cm, consisting of several materials, the values of the constants  $A$ ,  $D$ ,  $\alpha_n$  and  $\alpha_f$  vary along the wire. In the case of copper,  $A_{\text{Cu}} \approx 74 \text{ K A}^{-2} \text{ s}^{-1}$ ,  $D_{\text{Cu}} \approx 1.2 \text{ cm}^2 \text{ s}^{-1}$ , whereas in the case of a nickel-chromium alloy,  $A_{\text{NiCr}} \approx 4800 \text{ K A}^{-2} \text{ s}^{-1}$  and  $D_{\text{NiCr}} = 0.17 \text{ cm}^2 \text{ s}^{-1}$ . The values of  $\alpha_{n,f}$  barely depend on the precise metal, so we will simply take:  $\alpha_n \approx 330 \text{ s}^{-1}$  and  $\alpha_f \approx 30 \text{ s}^{-1}$ . Because of its much higher resistivity, the nickel-chromium alloy generates significantly more heat than copper. By alternating pieces of nickel-chromium (of size  $l_a$ ) and of copper (of size  $l_i$ ), one obtains a system which acts as a first approximation as the system studied in [6]. The system is then effectively described by:

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( D(x) \frac{\partial T}{\partial x} \right) + (A(x) I^2 - Q_-) \quad (5)$$

when  $A(x) = A_{\text{NiCr}}$  and  $D(x) = D_{\text{NiCr}}$  (respectively  $A_{\text{Cu}}$  and  $D_{\text{Cu}}$ ) depending on the nature of the metal at location  $x$ .

For the problem considered here, the values of  $\alpha$  and  $D$  allow us to construct a diffusive length scale,  $\mathcal{L}_T = (D/\alpha)^{1/2}$ . This length is estimated to be no larger than 1 mm. In practice, it is difficult to design an experiment with wires of sizes  $(l_a, l_i)$  significantly smaller than  $\mathcal{L}_T$ . This practical considerations suggests us to study the problem in the range of parameters  $l_i, l_a \gtrsim \mathcal{L}_T$ .

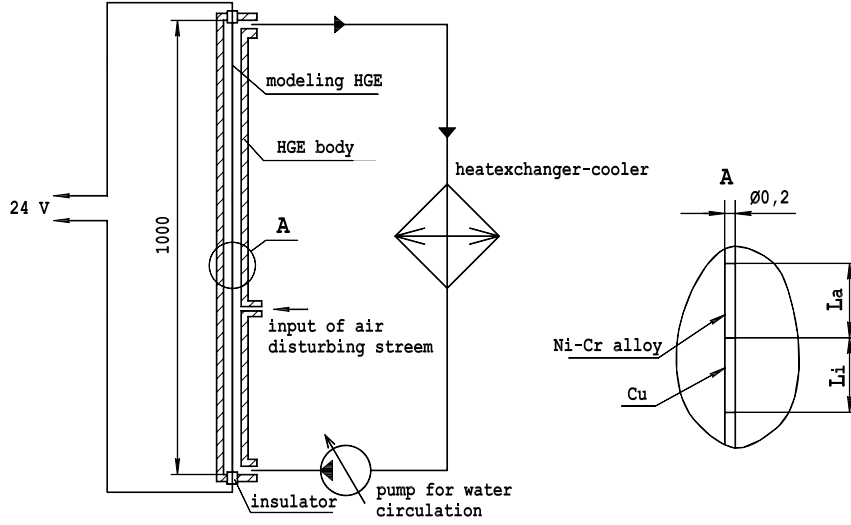
The particular case of an homogeneous system, where  $A(x) = A_{\text{NiCr}}$  is independent of  $x$ , is well-understood. The function  $Q_-(T)$  has the familiar inverted  $N$ -shape. For a range of values of  $Q_+ = A_{\text{NiCr}} I^2$ , the system has 3 roots, the largest and the smallest being stable, the intermediate one being unstable. Fronts separating the two stable values propagate with a velocity that vanishes at one particular value of  $Q_+$ , which is determined by the Maxwell construction. For  $Q_+ < Q_{+,c}$ , with  $Q_{+,c} \approx 11\,340 \text{ K s}^{-1}$  (corresponding to a current  $I_c \approx 1.54 \text{ A}$  with the numerical values chosen here), the nucleate boiling state prevails, whereas for  $Q_+ > Q_{+,c}$ , the film boiling state is observed.

A temperature dependent voltage difference is known to occur at the junction between two metals. The effect is weak, at most of the order of 1 mV, which is negligible compared to the voltage drop along  $\sim 1$  mm of nickel-chromium wire (diameter  $d = 0.1$  mm), with a current of  $\sim 1$  A running through it. In the rest of this paper, the voltage drop at the junction is neglected.

The choice of the 1-dimensional geometry is relevant to the important applications we have in mind. New interesting effects are expected in higher dimensions [11].

### 2.2 Numerical methods

To investigate the model problem, we have solve equation (4) using a straightforward Crank-Nicholson algorithm, second order in space and time and unconditionally stable [12]. The typical mesh size used in the calculations



**Fig. 1.** Proposed experimental setup to investigate the effects described in this work. A current runs through a heating wire, made of alternatively Ni-Cr alloy (active part) and Cu segments. The temperature of the fluid away from the heater is regulated.

is typically of the order of  $\Delta x = 10^{-2}$  mm; the time step is taken to be  $\Delta t = 10^{-3}$  s or less. These values ensure that the numerical results presented in this work remain unchanged when the mesh size and/or the time step are divided by 2, as we checked.

The numerical calculations carried out in this work correspond to a very long wire, of at least  $\sim 200$  periodicity lengths, with initially half of the system in the high temperature phase, the other half in the low temperature phase. No-flux (Neumann) boundary conditions are applied at the ends of the wire. The subsequent evolution of the system may lead to the invasion of the system by either of the two phases (low, high temperature), or to a steady front separating two distinct regions in the two different (low, high temperature) phases. This allows us to determine the phase diagrams, presented and discussed in this article.

### 2.3 Possible experimental realisation

Figure 1 illustrates our proposed experimental setup, aimed at studying the ideas developed in this article. The total wire has a length  $L = 1$  m, and consists of alternatively active (Ni-Cr alloys) and inactive (Cu) zones.

The wire model is placed in a transparent body, in order to visualize the thermal state of the HGE. Water is circulated by a pump and the heat produced by the HGE is removed by an exchanger-cooler. The thermal energy is produced by the Joule effect. The voltage applied may reach up to 24 V. In addition, local disturbances of the system may be applied, by using an air stream. Experimental results will be reported in a separate publication. Note that the interaction with the exchange cooler is not taken into account in the theoretical model studied here.

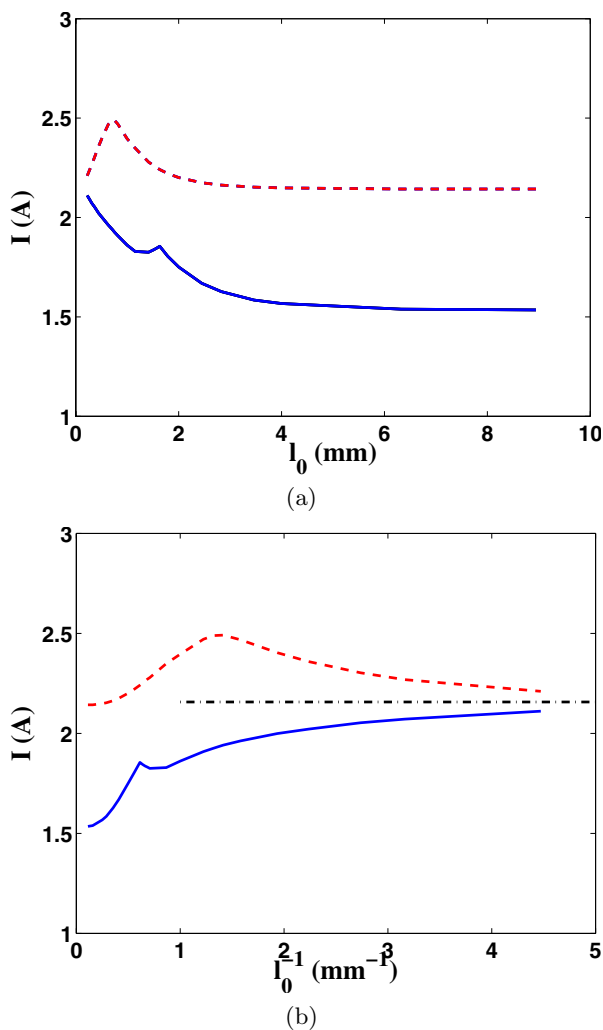
## 3 Results

In this section, we study the transition from nucleate to film boiling in the setup described above. Our calculations have been carried out by assuming a temperature of the fluid away from the heat source,  $T_0$ , to be at 20 °C, while the temperature where the transition from film to nucleate boiling is assumed to be at 120 °C.

### 3.1 Influence of the periodicity length

In the spirit of [6] we begin by considering the case of a system with  $l_a = l_i = l_0$ . Figure 2a shows as a function of  $l_0$  the two values of the current, above which film boiling prevails (upper curve) and below which nucleate boiling is observed (lower curve). In between the two curves, fronts separating regions at high temperature (film boiling) with regions at low temperature (nucleate boiling) stall. The same data is replotted as a function of  $1/l_0$ , Figure 2b. At large values of  $1/l_0$ , Figure 2b is very reminiscent of Figure 2 of [6], where the influence of the diffusion coefficient was considered over the stability of the system. In fact, elementary considerations show that changing  $l_0 \rightarrow \mu l_0$  at a fixed value of  $D$  is equivalent to changing  $D \rightarrow \mu^{-2} D$  at fixed  $l_0$ . As a result, diminishing  $l_0$  at a fixed value of  $D$ , as done here, amounts to increasing  $D$  at a fixed value of  $l_0$ , as in [6].

In the case of small value of  $l_0$ , the system behaves almost as an homogeneous system, which generates  $(A_{Cu} + A_{NiCr}) I^2/2$  per unit length. With the precise numerical values chosen here for the calculation of  $q_-$ , the critical value of the current for a strictly homogeneous system is found to be:  $I_0 = 2.157$  A corresponding to the horizontal dotted-dashed line shown in Figure 2b. This well understood regime, see [6], is in practice very difficult to observe experimentally, since the values of  $l_0$  are in fact

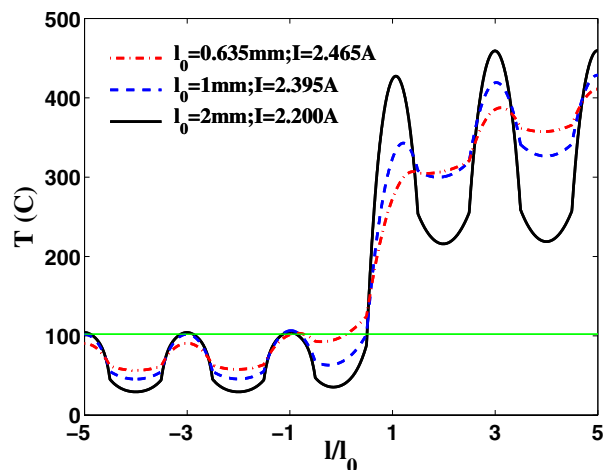


**Fig. 2.** Dependence of the currents limiting boiling regime (upper curve) and nucleate boiling (lower curve) in a system where  $l_a = l_i = l_0$ , as a function of the length  $l_0$  (upper figure, a) and of  $l_0^{-1}$  (lower figure, b)

smaller than  $\sim 1$  mm. The case of large values of  $l_0$  is more relevant for applications.

Figure 2a shows that when  $l_0$  becomes very large, the range of current where one observes stalled fronts separating the high and low temperature phases becomes independent of  $l_0$ . The limiting values of the current can be simply estimated by the following arguments.

When  $l_0$  is significantly larger than the diffusion length  $\mathcal{L}_T$ , a value of the current such that the heat generated  $Q_+$  is less than  $Q_{+,c}$ , corresponding to a steady front in a pure NiCr wire, cannot maintain a steady front between nucleate and front boiling. The value of the corresponding current is  $I_c \approx 1.54$  A. On the other hand, if  $I \gtrsim I_c$ , the heat generated is large enough to maintain film boiling throughout a section of the wire made of NiCr. However, this regime cannot extend through the Cu part of the wire. This argument shows that  $I = I_c$  is the limit for large values of  $l_0$  of the lower curve, below which the



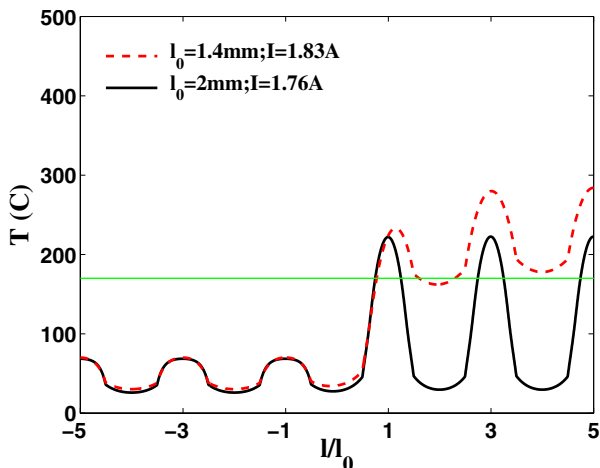
**Fig. 3.** Steady front solutions, obtained by solving numerically equation (4), separating a low temperature, nucleate boiling region (left) and a high temperature, front boiling region (right), for values of  $I$  slightly smaller than the value for which film boiling prevails. The maximum temperature in the nucleate boiling region is equal to the value of the temperature, for which the nucleate regime ceases to exist (indicated by the horizontal line).

low temperature phase (nucleate boiling) prevails. This is consistent with our own numerical results.

To understand the upper value of the current range, where steady fronts separate the high and low temperature phases, we consider three examples of fronts, computed numerically by solving equation (4) for the values of  $l_0 = 0.635$  mm, 1 mm and 2 mm and for values of  $I$  slightly smaller than the value of the current for which film boiling prevails, see Figure 3. Numerically, the transition is observed to occur when the largest value of the temperature in the low temperature phase is equal to the maximum value of the temperature before the nucleate boiling disappears (indicated by the horizontal line in Fig. 3). The mechanism of transition from nucleate to film regime when  $l_0$  is large is therefore due to a simple disappearance of the low temperature phase solution, and not to a change in front propagation velocity, as it is the case for small values of  $l_0$ .

In this respect, we note that the maximum of the upper curve in Figures 2a and 2b as a function of  $l_0$  actually corresponds to the limiting value, separating a regime where transition occurs via front propagation (for small values of  $l_0$ ), and a regime where transition is due to the disappearance of the nucleate state (for large values of  $l_0$ ).

The lower curves in Figure 2 also show a small cusp for a value of  $l_0$  close to 1.6 mm. Figure 4 shows numerically computed steady front solutions separating regions where nucleate boiling and film boiling regions for a value of  $I$  slightly larger than the value of  $I$  shown in Figure 2, both for a small ( $l_0 = 1.4$  mm) and for a large ( $l_0 = 2$  mm) value of the length. The difference between the two solutions in the high temperature phase (right side of the figure) can be clearly seen: at small values of  $l_0$ , the



**Fig. 4.** Steady front solutions, obtained by solving numerically equation (4), separating a low temperature, nucleate boiling region (left) and a high temperature, film boiling region (right), for values of  $I$  slightly higher than the value of the current below which nucleate boiling prevails, and for two values of  $l_0$ , on either side of the cusp in Figure 2. At small (large) enough length, the temperature in the inactive region on the hot side is larger (smaller) than the minimum value where film boiling exists, indicated by the horizontal line.

temperature in the inactive part of the system is larger than the minimum value of the temperature, indicated by the horizontal line, below which the film solution disappears. On the other hand, at larger values of  $l_0$ , the film solution cannot be maintained in the inactive region. This difference in behavior of the high temperature solution is at the origin of the cusp observed in the lower curve in Figure 2.

The main result of this subsection is that the stability of the wire, determined by the motion, or lack of motion, of fronts separating high and low temperature phases of boiling, is controlled by different mechanisms, depending on the value of the length  $l_0$ . When  $l_0$  is small compared to the thermal length,  $\mathcal{L}_T$ , thermal diffusion dominates, and the transition from film to boiling occurs by front propagation, as studied in [6]. At larger values of  $l_0$ , the stability of the system rests rather on the existence of various phases. This possibility had not been investigated in [6]. However, in view of the small value of the thermal length scale ( $\mathcal{L}_T \lesssim 1$  mm), experimentally realistic situations correspond to the latter case. Motivated by this remark, we consider the possible gain in safety in the model system, in conditions where the periodicity length along the wire is larger than the diffusion length.

### 3.2 Influence of the length of the inactive part

How does the length of the inactive part of the system affects the amount of heat that can be generated, the length of the active part being given, is the question we address in this subsection. We concentrate here on values of  $l_a$  practically accessible experimentally,  $l_a \gtrsim 1$  mm, that is,

on values of  $l_a$  which are larger than the values of the diffusion lengths.

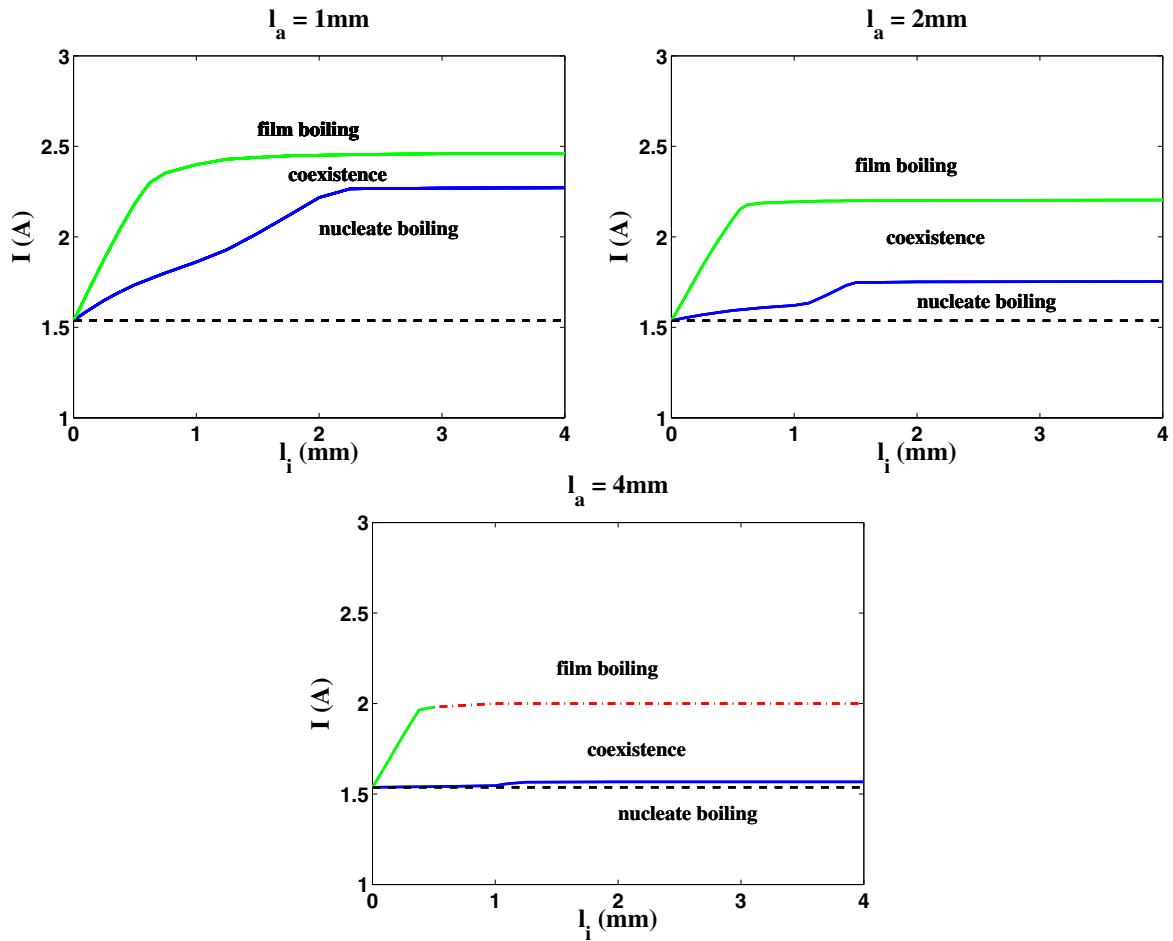
Figure 5 shows the results for three different values of the active length of the system:  $l_a = 1$  mm, 2 mm and 4 mm. The figures show, as a function of the length of the inactive part of the system,  $l_i$ , the value of the intensity of the current above which film boiling prevails (upper curve), and below which nucleate boiling dominates (lower curve). In between the two lines, coexistence between the two states is observed, in the sense that a front separating a region of high temperature, undergoing film boiling, and a region of low temperature, with nucleate boiling, is stalled. The horizontal dashed line corresponds to the value of the current  $I_c$ , corresponding to the separation between the two regimes for a uniform system, with  $l_i = 0$ .

The values of the current corresponding to the transition between the three regimes grow monotonously as a function of  $l_i$  for the three values of  $l_a$  studied. One observes that the smaller the value of  $l_a$ , the larger the values of the currents at which the transition occurs. In particular, at a value of  $l_a = 4$  mm, the value of the current below which nucleate boiling is stable is very close to  $I_c$ . However, a significant range of values of the current over which a front separating regions in the nucleate and in the film boiling is always observed. In the case of the large values of  $l_a$  ( $l_a = 4$  mm), an additional difficulty occurs. The maximum value of the temperature in the hot parts of the wire may reach very high values, where the wire would effectively melt. The part of the upper curve with the dashed-dotted pattern corresponds to a part where the hottest spots reach a temperature of 500 °C, where we estimate that irreversible damage would occur.

Figure 5 shows that by alternating active regions with a given size,  $l_a$ , one may safely increase the output per active length of the system. This is already a very positive result, since it shows that heat generation by the active parts of the wire can be maintained at a higher level than in an homogeneous system, without any risk of a transition to the film boiling, high temperature state with possibly dire consequences. Figure 5 demonstrates that the maximum heat released per unit length of the active part of the wire is  $\sim 75\%$  when the size of the active part is  $l_a = 4$  mm, and even higher at lower values of  $l_a$ .

An other way to characterize the heat generated by the system is to measure the power released per unit length in the system. In view of equation (5), the heat release per unit length in the system is simply:  $Q_{eff} \equiv (A_{NiCr} l_a + A_{Cu} l_i) I^2 / (l_a + l_i)$ . This quantity is to be compared with the maximal value of the heat generated by the system for a pure active system ( $l_i = 0$ ) in a state of nucleate boiling:  $Q_{+,c}$ . Figure 6 thus shows the value of  $Q_{eff}/Q_{+,c}$  as a function of  $l_i$  for the three values of  $l_a$ :  $l_a = 1$  mm, 2 mm and 4 mm.

These figures demonstrate that for the values of  $l_a$  chosen, there exists a range of values of  $l_i$ , where the heat generated per unit length is *larger* than the largest possible heat flux in a homogeneous system ( $l_i = 0$ ), in such a way that a region in the high-temperature, film boiling phase



**Fig. 5.** Stability of the boiling regime as a function of the inactive part length,  $l_i$ , the length of the active part of the wire being fixed:  $l_a = 1$  mm,  $l_a = 2$  mm and  $l_a = 4$  mm as indicated on each graph. The lower continuous line shows the maximum value of the current below which the nucleate phase is stable. The upper continuous curve shows the value of the current above which the high temperature, film boiling region entirely invades the system. In between the two lines, fronts separating the low and high temperature phases do not propagate. The horizontal dashed line shows the critical value of the current below which the nucleate boiling phase is stable in an homogeneous system ( $l_i = 0$ ). For the largest value of  $l_a$  (4 mm), the highest values of  $I$ , indicated by the dotted-dashed line, correspond to a temperature of the wire estimated to  $\sim 500$  °C, that would lead to melting of the wire.

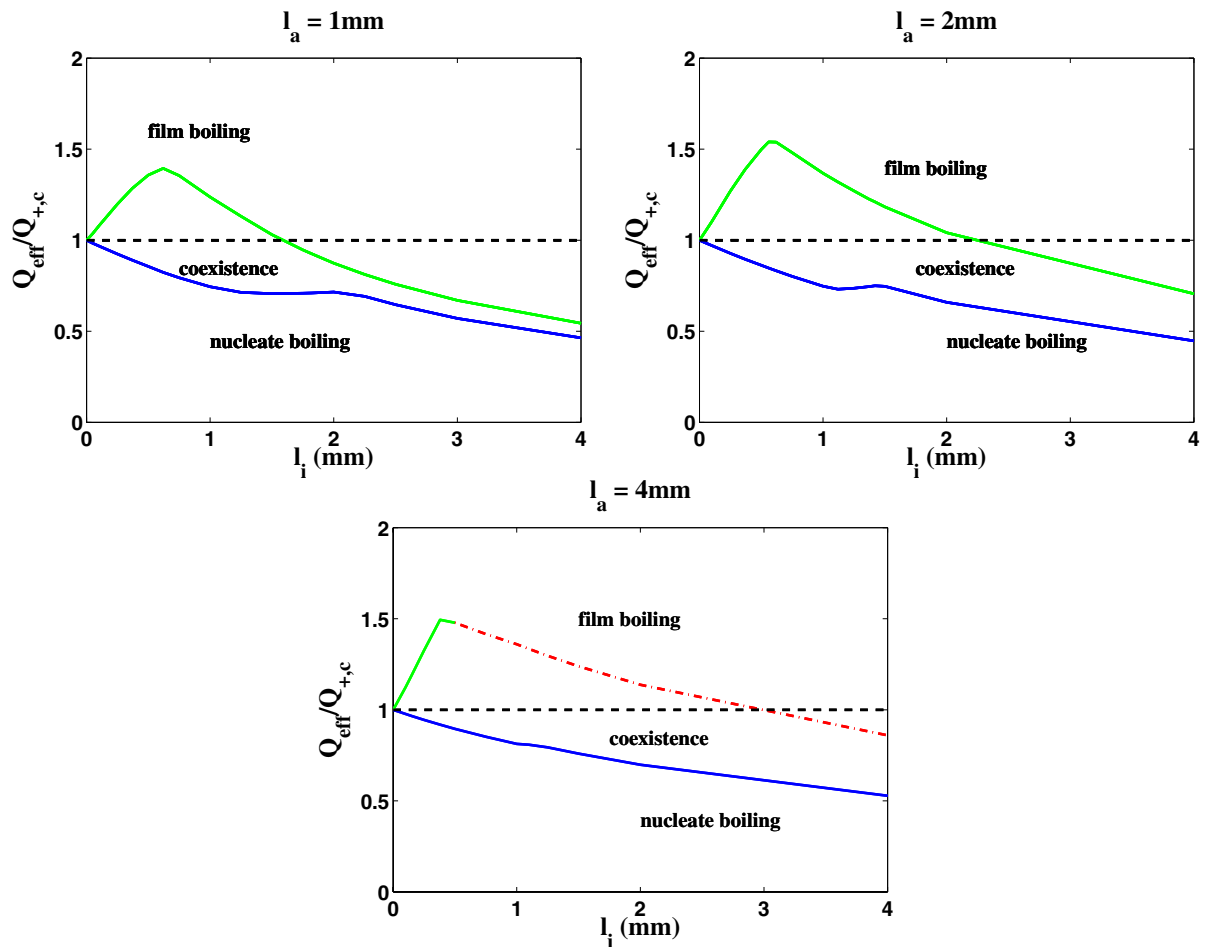
does not tend to invade regions in the nucleate boiling region. The range of values of  $l_i$  is limited: when  $l_i$  exceeds a certain value, the output per unit length becomes smaller than 1, implying that the system cannot generate more heat than an homogeneous system without being unstable, in the sense that a region where temperature would become high (transition to film nucleate boiling) would invade the whole wire. The largest value of  $l_i$  over which nucleate boiling is stable, and where the output per unit length of the system is larger than in the homogeneous case grows roughly proportional to the value of  $l_a$ .

As a conclusion of this subsection, we have demonstrated numerically that in the experimentally realistic case where the length of the active wire is large compared to the diffusion lengths, separating the heat generating elements by inactive parts does stabilize the nucleate boiling phase. The power output per unit length can even be *larger* than in an homogeneously heat generating config-

uration ( $l_i = 0$ ), by as much as  $\sim 40\%$ . In this sense, the predictions of [6] remains always valid, despite the different range of parameters, and the difference in the detailed mechanisms of transition between film and nucleate boiling.

## 4 Discussions

In this article, we have studied the transitions between nucleate and film boiling in a very simple model based on a plausible effective description of the heat removal, with a simple dependence on temperature. This approximation reduces the problem to a 1-dimensional reaction diffusion equation. We have considered a model systems consisting of heat generating sections, separated by passive regions, arranged periodically along the wire.



**Fig. 6.** Power output per unit of length of the wire,  $Q_{eff}$ , as a function the inactive part length,  $l_i$ , the length of the active part of the wire being fixed:  $l_a = 1$  mm,  $l_a = 2$  mm and  $l_a = 4$  mm as indicated on each graph. The value of  $Q_{eff}$  is normalized by the maximal value of the heat output per unit length,  $Q_{+,c}$ , in an homogeneous system ( $l_i = 0$ ). The lower continuous line shows the maximum value of  $Q_{eff}/Q_{+,c}$  in a system in the nucleate regime. The upper continuous line shows the power output above which boiling occurs only in the film phase. In between the two regions, fronts separating nucleate and film boiling regions do not propagate. For the largest value of  $l_a$  (4 mm), the higher value of  $l_i$  leads to a temperature of the wire estimated to  $\sim 500$  °C, that would lead to melting of the wire; the value of  $l_i$  for which this would happen is indicated by the dotted-dashed line.

One of our main conclusions is that the ratio between the periodicity length  $l_0$  along the wire and the characteristic thermal length scale  $\mathcal{L}_T$  strongly affects the mechanisms of transition between the fronts separating the high and low temperature phases of boiling. In the case of small values of  $l_0/\mathcal{L}_T$ , the temperature modulations are relatively weak, and the motion of the front, or lack thereof, determines the stability of the system. In the opposite case, where  $l_0/\mathcal{L}_T$  is larger than  $\sim 1$ , the stability of the system is determined by considerations of existence of the low or high temperature phases when a given current is applied.

Numerically, the value of the thermal length  $\mathcal{L}_T$  does not exceed 1 mm. We have also demonstrated that in the experimentally realistic case where the length of the active wire, typically a few millimeters, is larger than the diffusion length, alternating heat generating elements with in-

active elements results in a substantial stabilization of the nucleate boiling phase. One may even obtain an increase of the heat output per unit length, without any risk of transition to the high temperature film boiling regime.

Quantitatively, for the particular problem studied here, we obtain that (i) the maximum amount of power released per unit length of the active zone can be increased by 75% for elements of size  $l_a = 4$  mm, compared to a purely homogeneous system, and that (ii) the maximum amount of power per length can be increased by as much as  $\sim 40\%$ .

One obvious domain of application of the work described here concerns the problem of nuclear safety. The transition towards the high temperature phase can lead to a melt down of the core, with possibly very serious consequences. With this particular application in mind, the possibility to increase the power output per unit length of

the active zone without jeopardizing safety is of obvious interest. In this respect, our proposal consists in alternating nuclear material with inert material in fuel bars [13]. The design of a simpler experiment involving electric heating, such as the one proposed in this article, is an important step to validate the ideas put forward here.

Our work was supported in part by a grant from NATO under the Science for Peace and Security program.

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